

THE PION STRUCTURE FUNCTION IN A CONSTITUENT MODEL

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ABSTRACT

Using the recent relatively precise experimental results on the pion structure function, obtained from Drell–Yan processes, we quantitatively test an old model where the structure function of any hadron is determined by that of its constituent quarks. In this model the pion structure function can be predicted from the known nucleon structure function. We find that the data support the model, at least as a good first approximation.

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1 Introduction

In 1974 a model was proposed [1] for the deep inelastic scattering structure functions of a hadron in terms of constituent quarks with structure. For example, the proton is described in terms of three UUD constituents with an $SU(6)$ inspired wave function. At large Q^2 the virtual photon probes deep into one constituent and sees its parton structure. The proton structure function is obtained as a convolution of the Q^2 independent constituent wave function with the Q^2 dependent constituent structure function. Similar models of the nucleon in terms of constituents with structure have been considered over the years also to describe the static properties of nucleons [2, 3, 4]. In our case, the nucleon structure function is treated in full analogy with the case of Helium 3, with constituent quarks replacing nucleons. Of course one may object that nucleons, i.e. the constituents of Helium-3, are colourless and therefore can exist as unconfined units. On the contrary, the constituent quarks are confined, so that they cannot be really independent of each other and a colour field string must connect them to each other. However, it is conceivable that the content of the string in terms of sea partons and gluons could be small in comparison with the structure of the constituent. Alternatively, a string segment could be associated with the constituent in a universal way, independent of the constituent flavour and of the hadron, so that, in a sense, it becomes a part of the constituent itself. At the other extreme, the string could be responsible for the whole structure of the hadron. In this extreme case we would have a model of three structureless valence quarks and a sea of quark and gluons from the string [5] in principle different for different hadrons. The real hadron will probably be somewhat in between.

In this note we discuss the quantitative information that can be obtained on this issue from the available data on the pion structure function which have been collected from measurements of the Drell–Yan lepton pair production cross-section. In the proposed model where all the structure is in the constituents, one can start from the known parton densities in the nucleon, deconvolute the wave function and obtain the parton densities in the constituents. From these one can then predict the pion structure function, given a reasonable wave function for the pion. We will compare the predictions of this model with the data on the pion structure function obtained from the Drell–Yan process [6]. This kind of comparison has already been done long ago [4, 7] but the data on the pion structure function [8] are by now sufficiently precise to make the present re-analysis worthwhile. In fact the validity of the constituent-with-structure ansatz can now be significantly tested. We shall see that the model is in reasonable albeit not perfect agreement with the data.

It is true that the choice of the wave functions introduces some ambiguity in the prediction of the pion structure function from that of the proton. But there are important sum rules in this model that are valid independently of the wave function form. In fact the amount of momentum carried by gluons, by sea and by valence should be separately the same in the nucleon and in the pion at the same Q^2 (as a consequence of the fact that the constituents carry the totality of the hadron momentum). No such equality is predicted by the model where all the sea and gluons, or a substantial part of them, arises from the string, the structure of the string in the proton and in the pion being in principle different. The separate determination of sea and

gluons in the pion is difficult, because the available Drell–Yan data do not give any information on the pion structure functions at $x \leq 0.2$. Most of the information on the gluon distribution in the pion arises from the limited data on large p_T photons produced in π^+p reactions [8]. But the total momentum carried by sea and gluons is well determined being the complement to 1 of that of valence and one finds 0.61 ± 0.02 for the proton [9] and 0.54 ± 0.04 for the pion [8], at $Q_0^2 = 4 \text{ GeV}^2$. The results of this analysis appear to support to a fair degree of accuracy the constituents-with-structure model.

We recall that another application of the formalism of Ref. [1] is for nuclei. In Ref. [10], it is shown that a substantial part of the EMC effect (the A dependence of the nucleon structure functions) can be attributed to the distortion of the constituent wave function inside a nucleon due to the external nuclear field. The model can also be applied to polarized deep inelastic scattering [11]. In this model the constituents carry the totality of the proton spin and the observed *spin crisis* is described by a corresponding depletion of the fraction of the constituent spin which is carried by parton quarks. In this picture it is particularly clear that the experimental results are not at variance with the constituent model. Rather they have implications on the constituent structure.

2 The model

For definiteness consider a proton p or a positively charged pion π^+ . In the model where the structure of the hadron is due to the structure of the constituents, the parton density $r_h(x, Q^2)$ for a given parton type r in the hadron h is given by [1]:

$$r_h(x, Q^2) = \int_x^1 \frac{dz}{z} \left[U_h(z) r_U \left(\frac{x}{z}, Q^2 \right) + D_h(z) r_D \left(\frac{x}{z}, Q^2 \right) \right] , \quad (1)$$

where U_h is the density of up-constituents in the hadron $h = p, \pi^+$, D_p is the density of down-constituents in the proton p , while D_{π^+} is the density of \bar{D} (antidown) constituents in the pion π^+ , $r_{U,D}$ are the parton densities in the U or D constituents (for the pion r_D is actually $r_{\bar{D}}$ which is the same as \bar{r}_D). As the Q^2 evolution matrix does not depend on the target, i.e. it is the same for the partons in a proton or in a constituent, it follows that if the convolution is valid at one Q^2 it will remain valid at all Q^2 . Note that the moments

$$r_h^{(n)}(Q^2) = \int_0^1 dx x^{n-1} r_h(x, Q^2) \quad (2)$$

are simply given by a sum of products of moments:

$$r_h^{(n)}(Q^2) = U_h^{(n)} r_U^{(n)}(Q^2) + D_h^{(n)} r_D^{(n)}(Q^2) . \quad (3)$$

For short hand we indicate the above convolution by

$$r_h = [U_h \otimes r_U + D_h \otimes r_D] . \quad (4)$$

For example, the gluon density in the proton is given by

$$g_p = [U_p \otimes g_U + D_p \otimes g_D] = (U_p + D_p) \otimes g_U \quad (5)$$

where the last step is due to the equality of the gluon density in U and D constituents. Actually it is important to note that, by using obvious isospin relations (like $u_D = d_U$, etc.), for all partons kinds r_p one can refer to the densities of partons in the U constituent. This is also true in the pion case. Now recall that the first moment of U and D are $U_p^{(1)} = 2, D_p^{(1)} = 1$ while, for the second moments, $U_p^{(2)} + D_p^{(2)} = 1$, because constituents carry the totality of charge and momentum of the proton. Hence $g_p^{(2)} = g_U^{(2)}$. Clearly, for similar reasons, also $g_\pi^{(2)} = g_U^{(2)}$. Thus, independent of the wave functions, the total momentum of gluons in p and in π^+ are predicted to be the same at the same Q^2 . By an identical argument, the same prediction holds for the total sea second moment and consequently for the total momentum carried by valence.

3 Parameters of the model

In order to predict the parton densities in the pion from those in the proton, we take the proton parton densities given by the most recent fits of all available data obtained by Martin et al. in Ref. [9]. Precisely we use the set of parton densities labeled by MRS(G), with the two-loop Q^2 evolution evaluated for $\Lambda = 255$ MeV, where Λ refers to $N_f = 4$ in the \overline{MS} definition, corresponding to $\alpha_s(m_Z) = 0.114$. Other available sets of structure functions will be used to check the stability of the results (also with different values of Λ). As for the distributions of the U and D constituents in the proton we take those given in Ref. [1]. These constituents densities, based on $SU(6)_W \otimes O(3)$ at $p_z \rightarrow \infty$, are complicated and will not be reproduced here (it suffices to say that the parameters introduced in Ref. [1] are fixed to the values $\beta = 0.44$, $a^2 = 0.8$). Simpler choices would also work and we have checked that no important changes in the final results are obtained with different starting wave functions. Then for the parton densities in the U constituent we adopt the following parametrisation at $Q_0^2 = 4$ GeV²

$$\begin{aligned} q_U(x, Q_0^2) &= C_s \frac{(1-x)^{(D_s-1)}}{x^{(1+d_s)}} + \delta_{uq} \frac{B^{-1}(A, \frac{1}{2})(1-x)^{(A-1)}}{\sqrt{x}}, \\ g_U(x, Q_0^2) &= C_g \frac{(1-x)^{(D_g-1)}}{x^{(1+d_g)}}. \end{aligned} \quad (6)$$

Where $B(x,y)$ is the Euler beta function. In the quark formula there is a valence term, only present for the u parton quark, and a universal sea term. The differences in the sea composition (strange vs. non-strange, \bar{u} vs. \bar{d} etc.) are irrelevant here and have been neglected. At small x , a stronger behaviour than $1/x$ for sea and gluon densities, parametrised by the positive coefficients d_s and d_g , has been allowed according to the results obtained by ZEUS and H1 at HERA. Of course the momentum sum rule imposes a relation among the parameters.

Figure 1: Proton parton densities at $Q_0^2 = 4 \text{ GeV}^2$. Solid lines refer to the fit of MRS(G) [9]; dashed lines refer to the present model. The differences are well within the uncertainties of the MRS(G) fit.

The parameters appearing in the above formulae were fitted to reproduce the input parton densities in the proton, given the chosen wave function. The values of the parameters obtained from the fit are: $A = 0.776$, $C_s = 0.5$, $D_s = 3.3$, $d_s = 0.085$, $D_g = 1.3$, $\delta_g = 0.45$. The comparison between the input parton distributions of Ref. [9] at Q_0^2 and the results of the fitted distributions for the model are shown in Fig. 1. As seen, given the accuracy to which the parton densities are known, a very good fit is obtained. Thus there is no doubt that the proton data are nicely consistent with the model.

For the total momentum fraction carried by valence in the proton at Q_0^2 one has:

$$V_p^{(2)}(Q_0^2) = 0.39 \pm 0.02 \quad (7)$$

where $V = [u - \bar{u} + d - \bar{d}]$. The error has been estimated by reevaluating the moment starting from the available recent compilations of the proton parton densities, as shown in Table 1. We also varied the value of Λ in a range corresponding to $0.110 \leq \alpha_s(m_Z) \leq 0.125$ using the recent

	$\alpha_s(m_Z)$	$v_p^{(2)}(Q_0^2)$	$g_p^{(2)}(Q_0^2)$
CTEQ1M	0.111	0.390	0.419
MRSS0	0.110	0.386	0.448
MRSD-	0.110	0.383	0.444
MRS(G)	0.114	0.392	0.427
MRS 110	0.110	0.377	0.434
MRS 115	0.115	0.382	0.428
MRS 120	0.120	0.390	0.421
MRS 125	0.125	0.398	0.414

Table 1: Second moment of valence, $v_p^{(2)}$, and gluon, $g_p^{(2)}$, in the proton at $Q_0^2 = 4 \text{ GeV}^2$ for some of the most recent parton density parametrizations.

results of Ref. [12]. The total error is a combination of the uncertainty at fixed α_s with that from varying α_s . The difference in $V_p^{(2)}$ using either the input nucleon densities or the fitted densities in the U constituent is completely negligible given the quoted error.

4 Results

Having derived the parton densities in the U constituent we now proceed to predict the pion structure functions. The only ingredient which is still needed is the distribution of U and \bar{D} constituents in the pion. Following Ref. [7] we take

$$\begin{aligned}
 U_{\pi^+} &= \bar{D}_{\pi^+} = 1/2V_{\pi^+}, \\
 V_{\pi^+}(x) &= \sqrt{\frac{8\tilde{\beta}}{\pi}} \frac{1}{x(1-x)} \exp[-2\tilde{\beta} \ln^2 \frac{x}{1-x}],
 \end{aligned} \tag{8}$$

where the parameter $\tilde{\beta}$, is fixed to the value $\tilde{\beta}=0.1$ in such a way as to approximately have $x \cdot v_\pi(x, Q_0^2) \sim (1-x)$ as $x \rightarrow 1$, according to the Drell–Yan–West relation [13], and v_π is the valence quark distribution in the pion.

The predictions for the parton densities in the pion at Q_0^2 are simply obtained by convoluting according to eqs. 1, the constituent distributions in eq. 8 with the parton densities in the constituent specified in eqs. 6 as a result of the proton fit. In particular, as already mentioned, the predicted second moment of valence coincides with the result for the proton and is given in eq. 7.

The above predictions should now be confronted with the experimental data from Drell–Yan processes [6]. The way to extract the pion structure functions from the Drell–Yan data has been recently discussed in Ref. [8]. As it is well known the Drell–Yan cross-section is obtained by a convolution of the parton densities in the proton times those in the pion times the partonic cross-section. The latter includes the QCD correction which leads to a quite substantial K

factor [14] at the relevant dimuon mass scale (typically between the J/ψ and the Υ). Thus in principle in order to extract the pion densities one has to compute the K factor. The authors of Ref. [8] chose to write the K function in the form $K(x_F, Q^2) = K^{(1)}K'$, where $K^{(1)}$ is the simple K factor computed at one loop accuracy, while K' includes the effect of higher orders (including the correction for a possible bad choice of α_s in the leading term). In Ref. [8] K' was fitted from the data. This procedure is only justified if K' is really a constant in x_F . Indeed the fit is sensitive to a constant rescaling of V . In fact the valence is the dominant contribution at the rather large values of x where the data for the pion structure function exist and the overall scale of valence is normalized by its first moment $V_\pi^{(1)} = 2$, where $V_\pi = u + \bar{d}$. A consistency check is that K' , arising from higher orders, should come out reasonably close to 1. In the fits of Refs. [8] K' ends up in a range between $1.1 \div 1.3$.

If indeed K' is with good approximation a constant, we can take the results of the fits in Ref. [8] as a compact description of the actual data. Recently an almost complete calculation of the rapidity dependence of the K factor at two loop accuracy has been performed in Ref. [15]. This calculation is not complete because the effect of soft gluon contributions (within a specified definition) is not included. We have repeated the procedure of Ref. [8] with the available two-loop QCD corrections to the Drell–Yan x_F differential cross-section. We found that with a very good accuracy K' is indeed a constant over the rapidity range of the experiment. Thus we can validate the procedure of Ref. [8]. Of course further uncertainties on the result of the fit beyond the statistical accuracy arise from the rudimentary parametrization adopted for the pion densities and from the assumptions made for the sea and gluon densities which the data do not much constrain.

In Fig. 2 we present a comparison between the model fit of the data and the best fit obtained in Ref. [8]. The theoretical predictions are presented with and without inclusion of the K' factor. We see that the model fits the data quite well but at the price of a somewhat larger K' . While in the model independent fit K' ranges between 1.1 and 1.3, in the model one needs a larger K' , $K' = 1.3 \div 1.6$.

Clearly the model independent fit has a slightly better χ^2 than the model. Also, in the model, the resulting values of the K' factor are a bit too large to be really satisfactory. This difference of K' factors is a consequence of the different values for the second moment of valence found in the model independent fit and for that implied by the model:

$$V_\pi^{(2)}(4 \text{ GeV}^2) = 0.46 \pm 0.04 \text{ (fit)} \quad (9)$$

$$V_\pi^{(2)}(4 \text{ GeV}^2) = 0.39 \pm 0.02 \text{ (model)} \quad (10)$$

where, of course, the model value is the same as in the proton. The error attributed to the fit is our estimate which takes into account the error within the procedure of Ref. [8], as given by the authors, plus the ambiguities related to the assumptions made in the procedure (mainly from the parametrisation choice for the pion, the x_F independence of K' , the value of α_s , etc.). A plot of the resulting structure function of the pion, in the present model, is shown in Fig. 3, where it is compared with the fit of [8].

Figure 2: A comparison of a sample of experimental Drell–Yan data (π^-W reactions) with the fit in Ref. [8] (solid lines) and with the present model (dashed lines). In each case we present result with (upper curves) and without (lower curves) the K' factor. Thus both sets of curves can fit the data rather well but the model needs a larger K' factor.

5 Conclusions

In conclusion, the constituent-with-structure model is shown to provide a reasonably accurate description of the pion structure functions as determined by experiment. This is particularly remarkable in that the pion is a very peculiar hadron with mass that vanishes in the chiral limit. Thus there is a strong indication that the model can actually provide a reasonable first approximation of the structure functions of any other hadron for which no data exist.

Finally, we recall that the second and third moments of the valence parton densities in the pion have been estimated in lattice QCD in the quenched approximation [16]. There the result for the second moment was $V_\pi^{(2)}(49 \text{ GeV}^2) = 0.46 \pm 0.07$ which corresponds to $V_\pi^{(2)}(4 \text{ GeV}^2) = 0.55 \pm 0.08$. This is a rather large value in comparison with the prediction

Figure 3: The resulting valence distribution $x \cdot v_\pi(x)$ in the model (solid line) compared with the fit of Ref. [8] given as a band that includes the uncertainties on their procedure.

of the model. However, it is known that quenched lattice calculations fail to reproduce the momentum fractions carried by up and down quarks in the proton [17]. For the proton the lattice results are larger than the fitted values by at least a factor of two. Thus the conclusion is that the quenched approximation appears to be rather poor for the calculation of hadronic structure functions.

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